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#### ABSTRACT

The instability and the criteria for decomposition of a viscuous fuel film in air, based on consideration of the unstable waves arising on the surface of separation, are established.

With the jet process of fuel consumption, the course of fuel atomization and combustion depends, to a significant extent, on the injection process.

The problem of correct fuel injection was one of the most difficult problems in the development of high-speed diesels. Even today, the injection problem is still not completely solved.

In theoretical considerations of this question, the investigation is ordinarily limited to the disruption of fuel jets, using the method of small oscillations. G. I. Petrov and T. D. Kalinina (ref. 1) in this manner established the stability conditions for circular jets of nonviscous fuel.

The attempt is made below to establish the instability and the criteria for decomposition of a viscous fuel film in air, based on consideration of the unstable waves arising on the surface of separation.

The criteria of similarity, derivable from the equation of ultimate stability of the surface of separation, may also be applied to the investigation of the decomposition and injection of planar and tubular fuel jets.

Let the origin of coordinates be at the output of the port. The x axis is directed along the undisturbed surface of the fuel, while the y axis is perpendicular to this surface.

The equations of the nonstationary motion of the viscous, nonconsumed fuel, ignoring gravitational forces, have the form

$$\frac{\partial U}{\partial t} + (U \text{ grad}) U = -\frac{1}{\rho_m} \text{ grad } P_m + v_m \Delta U, \qquad (1)$$

subscript 
$$m = f$$
 (fuel) div  $U=0$ , (2)

where U is the fuel velocity vector;  $\rho_{\mathbf{f}}$  is the fuel density;  $\nu_{\mathbf{f}}$  is the fuel kinematic coefficient of viscosity and,  $P_{\mathbf{f}}$  is the pressure in the undisturbed fuel.

We now assume that the surface of the liquid fuel is subjected to disturbances. Disturbances on the surface may occur for a number of reasons: irregularities of the walls of the discharge ports, roughness of the walls, turbulence of flow, compression and expansion of the fuel in its passage through the injection apparatus, etc.

As the result of the applied disturbances, the components of motion are subjected to perturbations

$$U_x = U_c + u_x; U_y - u_y; P_m = P + p_1,$$
 (3)

where  $\mathbf{U}_{\mathbf{c}}$  is the constant fundamental velocity of fuel motion.

The new components of motion must also satisfy equations (1) and (2).

Substituting the values of the components of motion from (3) in equations (1) and (2), and ignoring both second order terms and convective terms, since the amplitudes of the waves under consideration are very small in comparison with the wavelengths, we obtain the equations of the superimposed disturbances in coordinate form

$$\frac{\partial u_{x}}{\partial t} = -\frac{1}{\rho_{m}} \frac{\partial p_{1}}{\partial x} + \nu_{m} \Delta u_{x}, \qquad (4)$$

$$\frac{\partial u_{y}}{\partial t} = -\frac{1}{\rho_{m}} \frac{\partial p_{1}}{\partial y} + \nu_{m} \Delta u_{y}, \qquad (5)$$

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0. \tag{6}$$

By virtue of the continuity equation, the components of velocity of planar flow can always be expressed in terms of the flux in the following way

$$u_x = \frac{\partial \psi}{\partial y}; \quad u_y = -\frac{\partial \psi}{\partial x}.$$
 (7)

If we substitute these values of the velocity components in equations (4) and (5), we obtain

$$\frac{\partial^2 \psi}{\partial t \, \partial y} = -\frac{1}{\rho_m} \frac{\partial \rho_1}{\partial x} + \nu_m \, \Delta \, \frac{\partial \psi}{\partial y} \tag{8}$$

$$-\frac{\partial^2 \psi}{\partial t \, \partial x} = -\frac{1}{\rho_m} \frac{\partial p_i}{\partial y} - \nu_m \, \Delta \, \frac{\partial \psi}{\partial x}. \tag{9}$$

We now eliminate pressure  $p_1$  from these equations by differentiating equation (8) with respect to y, differentiating equation (9) with respect to x, and subtracting, thus obtaining

$$\frac{\partial \Delta \dot{\gamma}}{\partial t} = v_{\rm m} \ \Delta \Delta \dot{\gamma}. \tag{10}$$

This equation decomposes into two equations

$$\Delta \dot{\gamma}_1 = 0, \tag{11}$$

$$\frac{\partial \psi_2}{\partial t} = \nu_{\rm m} \Delta \psi_2. \tag{12}$$

It is well known that, for plane motion, both the flux function and the velocity potential satisfy the Laplace equation, so that equation (11) can be replaced by the equation

$$\Delta \varphi_{\rm m} = 0, \tag{13}$$

where  $\phi_f$  is the fuel velocity potential.

For the two-dimensional problem, equations (12) and (13) are written in expanded form as

$$\frac{\partial \psi_2}{\partial t} = v_m \left( \frac{\partial^2 \psi_2}{\partial x^2} + \frac{\partial^2 \psi_2}{\partial y^2} \right), \tag{14}$$

$$\frac{\partial^2 \varphi_{\rm m}}{\partial x^2} + \frac{\partial^2 \varphi_{\rm m}}{\partial y^2} = 0. \tag{15}$$

In accordance with the wave character of the phenomenon, we write the solution of these equations in the form of periodic functions of coordinate x and time t:

$$\psi_2 = F_1 \ (y) e^{i (\kappa x - \omega t)},$$
 (16)

$$\varphi_{m} = F_{2} (y) e^{i (\kappa x - \omega t)}, \qquad (17)$$

where k is the spatial angular velocity of oscillation (the wave number), related to the wave length 1 by the relationship  $k = 2\pi/1$ , and  $\omega$  is the complex frequency of oscillation with time.

By substituting the solutions in (16) and (17) into equations (14) and (15), we find the following equations for the functions  $F_1(y)$  and  $F_2(y)$ 

where

and

whence

$$F_{1}^{"}(y) - \lambda^{2} F_{1}(y) = 0,$$

$$\lambda^{2} = k^{2} - \frac{i\omega}{v_{m}}$$

$$F_{2}^{"}(y) - k^{2} F_{2}(y) = 0,$$

$$F_{1}(y) = B_{1} e^{\lambda y} + B_{2} e^{-\lambda y}$$

$$F_{2}(y) = A_{1} e^{\kappa y} + A_{2} e^{-\kappa y}.$$

We thus obtain, for  $\psi_{\mathcal{O}}$  and  $\varphi_{\mathbf{f}}$ ,

$$\psi_2 = (B_1 e^{xy} + B_2 e^{-xy}) e^{t (\kappa x - \omega t)},$$
 $\Psi_m = (A_1 e^{\kappa y} + A_2 e^{-\kappa y}) e^{t (\kappa x - \omega t)}.$ 

Since we are considering only surface waves, in which the range of motion 1 is small in comparison with the depth, this latter can be considered as large as is desired. Then, from physical considerations, it is clear that, as  $y = -\infty$ , the velocity of wave motion must reduce to zero.

This condition is satisfied if we set the constants  $A_9 = 0$  and  $B_9 = 0$ .

The solution then takes the form

$$\psi_{\mathbf{s}} = \mathbf{B}_{i} \ \mathbf{e}^{\lambda \mathbf{y} + \mathbf{i} \ (\kappa \mathbf{x} - \omega \mathbf{i})}, \tag{18}$$

$$\varphi_{in} = A_i e^{\kappa y + 1 (\kappa x - \omega i)}. \tag{19}$$

To determine the total pressure in the jet, we use the Lagrange integral for an ideal fluid, since it is clear, from physical considerations, that the appearance of viscosity in a fluid changes the value of the frequency, but not the pressure distribution.

$$\frac{P_{\rm m}}{r_{\rm m}} = -\frac{\partial \varphi_{\rm m}}{\partial t} - \frac{1}{2} \left[ \left( U_{\rm c} + \frac{\partial \varphi_{\rm m}}{\partial x} \right)^2 + \left( \frac{\partial \varphi_{\rm m}}{\partial y} \right)^2 \right] + F(t), \tag{20}$$

where F(t) is an arbitrary function of time. This latter may always be set equal to zero.

If we omit all second order terms, as well as terms of no consequence for the question at issue, we can write equation (20) in the form (ref. 2)

$$\frac{P_{m}}{\rho_{m}} = -\frac{\partial \rho_{m}}{\partial t} - U_{c} \frac{\partial \rho_{m}}{\partial x}. \tag{21}$$

Substituting the value of the velocity potential,  $\phi_{\text{f}},$  we obtain

$$P_{iii} = \rho_{iii} \left( k U_c - \omega \right) i \beta_1 e^{ky + i \left( kx - \omega i \right)}. \tag{22}$$

The spreading of capillary waves on the surface of the moving fuel will give rise to the appearance of analogous waves in the ambient air as well.

The equations of the oscillations of the air close to the jet surface, ignoring convection terms, gravitational forces, and viscous forces, are written in the form

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{2^{10}} \text{ grad } \mathbf{p}_2, \tag{23}$$

subscript 
$$B = a (air)$$
  $\partial iv v = 0$ , (24)

where v is the disturbed air velocity vector;  $\rho_a$  is air density and,  $p_2$  is the pulsation pressure in the air medium.

Equation (23) shows that the motion of the superimposed disturbances of the air medium is potential, so that for it we may write

$$v = \operatorname{grad} \varphi_{B}$$
 (25)

The air velocity potential  $\phi_{\mathbf{a}}$  satisfies the equation

$$\Delta \varphi_{_{\rm B}} = 0 \tag{26}$$

or, for the two-dimensional problem

$$\frac{\partial^2 \varphi_{\mathbf{B}}}{\partial x^2} + \frac{\partial^2 \varphi_{\mathbf{B}}}{\partial y^2} = 0. \tag{27}$$

The solution of equation (27), analogously to that of equation (15), is written in the form

$$\varphi_{_{\rm B}}=F_{_{\rm 3}}$$
 (y)  ${\rm e}^{i~(\kappa x~-\omega i)}.$ 

We have

$$F_3''(y) = K^T F_3(y) = 0,$$
  
 $F_2(y) = C_1 e^{KY} + C_2 e^{-KY},$ 

from where

and

$$\varphi_{\rm B} = (C_1 \, {\rm e}^{{\rm K} y} + C_2 \, {\rm e}^{-{\rm K} y}) \, {\rm e}^{{\rm f} \, ({\rm K} x - \omega t)}.$$

As  $y \to \infty$ , the velocity potential must tend to zero. This is possible if we set  $C_1 = 0$ .

We therefore obtain

$$\varphi_{B} - C_{2} e^{-\kappa y + i (\kappa x - \omega t)}. \tag{28}$$

The pressure in the ambient air is easily determined from equation (21) by introducing in it the air density, the velocity potential  $\phi_a$ , and the velocity of air motion  $U_a$ .

We have

$$\frac{P_{B}}{\rho_{B}} = -\frac{\partial \varphi_{B}}{\partial t} - U_{B} \frac{\partial \varphi_{B}}{\partial x},$$

so that

$$P_{B} = \rho_{B} (k U_{B} - \omega) i C_{2} e^{-\kappa y + i (\kappa x - \omega!)}.$$
 (29)

We denote by h the height of the raised disturbed surface above the undisturbed one, and we assume that the rise of the surface is a periodic function of  ${\bf x}$  and  ${\bf t}$ 

$$h = He^{i (\kappa x - \omega t)}. \tag{30}$$

If we now denote by  $\alpha$  the surface tension, then the pressure engendered by the surface tension will equal (ref. 3)

$$P_{\alpha} = \alpha \, \frac{\partial^2 h}{\partial x^2}$$

or

$$P_{,\alpha} = -\alpha k^2 H e^{i(\kappa x - \omega t)}. \tag{31}$$

The following condition must hold on the free surface of the fluid

$$\left[-P_{\rm m} + 2\mu_{\rm m} \frac{\partial}{\partial y} \left(\frac{\partial z_{\rm m}}{\partial y} - \frac{\partial^2 z_{\rm s}}{\partial x}\right)\right]_{y=0} = P_{x}, \tag{32}$$

where  $\mu_{\hat{\mathbf{f}}}$  is the fluid dynamic coefficient of viscosity. Substituting the values for the quantities in equation (32), we obtain

$$-\rho_{\rm m} (k U_{\rm c} - \omega) i A_1 + 2 \mu_{\rm m} (k^2 A_1 - ik\lambda B_1) - \rho_{\rm m} (k U_{\rm B} - \omega) i C_2 = -\alpha k^2 H.$$
(33)

To determine the constants  $A_1$ ,  $B_1$ ,  $C_2$  and H, we use the supplementary boundary conditions, expressing the absence of skipping and the continuity of the tangential forces on the surface of separation.

We have

$$\frac{\partial h}{\partial t} + U_{c} \frac{\partial h}{\partial x} = \left(\frac{\partial \hat{\tau}_{m}}{\partial y} - \frac{\partial \psi_{2}}{\partial x}\right)_{y=0}, \tag{34}$$

$$\frac{\partial h}{\partial t} + U_B \frac{\partial h}{\partial x} = \left(\frac{\partial \tau_B}{\partial y}\right)_{y=0}, \tag{35}$$

$$\mu_{\rm m} \left[ \frac{\partial}{\partial y} \left( \frac{\partial \varphi_{\rm m}}{\partial x} + \frac{\partial \varphi_{\rm 2}}{\partial y} \right) + \frac{\partial}{\partial x} \left( \frac{\partial \varphi_{\rm m}}{\partial y} - \frac{\partial \varphi_{\rm 2}}{\partial x} \right) \right]_{\rm y=0} = 0. \tag{36}$$

After substitution of h,  $\phi_{\hat{I}},\,\psi_2$  and  $\phi_a$  in equations (34), (35) and (36), respectively, we obtain

$$iH (k U_c - \omega) = kA_1 - \lambda iB_1, \tag{37}$$

$$iH(k U_B - \omega) = -kC_2, \tag{38}$$

$$2i k^{\circ} A_{1} + (\lambda^{2} + k^{2}) B_{1} = 0.$$
(39)

We determine H from equations (37) and (38), and then, by equating the expressions thus obtained, we find the constant  $\mathbf{C}_2$ 

$$C_2 = -\frac{(\kappa A_1 - \lambda i B_1) I (\kappa U_8 - \omega)}{i\kappa (\kappa U_c - \overline{\omega})}.$$

We determine the constant  $B_1$  from equation (39)

$$B_i = -\frac{2i \kappa \Lambda_i}{\lambda^2 + \kappa^2}.$$

We use (37) for the constant H

$$H = \frac{\kappa A_1 - \lambda i B_1}{i (\kappa U_c - \omega)}.$$

By now substituting the values found for  ${\rm C_2}$ ,  ${\rm B_1}$  and  ${\rm H}$  in equation (33), we obtain

$$-\rho_{\rm m} \left(k U_{\rm c} - \omega\right) i + 2\mu_{\rm m} \left(k^2 + ik\lambda \frac{2i \kappa^2}{\lambda^2 + \kappa^2}\right) -$$

$$-\rho_{\rm B} \left(k U_{\rm B} - \omega\right)^2 \frac{\left(\kappa + \lambda i \frac{2i \kappa^2}{\lambda^2 + \kappa^2}\right)}{i\kappa \left(\kappa U_{\rm c} - \omega\right)} = -\alpha k^2 \frac{\left(\kappa + \lambda i \frac{2i \kappa^2}{\lambda^2 + \kappa^2}\right)}{i\left(\kappa U_{\rm c} - \omega\right)}.$$

We introduce the notation  $D = K - 2\lambda \frac{\kappa^2}{\lambda^2 + \kappa^2}$ 

and will then have

$$k\rho_{\rm m} (k U_c - \omega)^2 + 2\mu_m k^2 Di (k U_c - \omega) - \rho_B (k U_B - \omega)^2 D = -\alpha k^2 D.$$
 (40)

Equation (40) allows us to determine the variation of the frequency with time.

With the notation

$$k U_B - \omega = \omega_i$$
,

we have

$$k U_c - \omega = k (U_c - U_B) + \omega_I$$

and equation (40) assumes the form

$$k\rho_{m} [k(U_{c} - U_{B}) + \omega_{I}]^{2} + 2\mu_{m} k^{2} Di [k(U_{c} - U_{B}) + \omega_{I}] - \rho_{B} \omega_{I}^{2} D + \alpha k^{3} D = 0$$

or

$$\begin{array}{ll} (k\rho_{m}-\rho_{B}\,D)\,\omega_{1}^{2}+2k^{2}\,[\rho_{m}\,(U_{c}\,-U_{B}\,)-\mu_{m}\,Di]\omega_{1}+k^{8}\,\rho_{m}(U_{c}\,-U_{B}\,)^{2}+\\ &+2\mu_{m}\,k^{8}\,D\,i\,(U_{c}\,-U_{B}\,)\,+\alpha\,k^{3}\,D\,=0, \end{array}$$

from where

$$\omega_{1} = \frac{-\frac{k^{2} \left[ \gamma_{in} \left( U_{c} - U_{B} \right) + \mu_{m} D_{I} \right] + \mu_{m} D_{I} \right] + \mu_{m} \left[ \rho_{m} - \rho_{B} D_{I} \right]}{k \rho_{m} - \rho_{B} D_{I}}$$

$$\frac{\sqrt{\frac{k^{4} \left[ \rho_{m} \left( U_{c} - U_{B} \right) + \mu_{m} D_{I} \right]^{2} - \left( k \rho_{m} - \rho_{B} D_{I} \right) \left[ k^{3} \rho_{m} \left( U_{c} - U_{B} \right)^{2} + \mu_{m} \left( V_{C} - V_{B} \right)^{2} + \mu_{m} \left[ V_{C} - V_{B} \right] + \mu_{m} \left[ V_{C} - V_{C} \right] +$$

We denote the expression under the radical sign by  $N^2$ . The stability of the fuel surface will depend on the sign of the expression under the radical sign.

For  $\mathbb{N}^2 > 0$ , the surface will retain the conditions for stability, the surface tension tending to obliterate deviations.

For  $N^2 < 0$ , the surface of separation will be unstable. We obtain the boundary separating the region of stable partial solutions from the unstable ones from the condition  $N^2 = 0$ , which leads to the equation

$$k^{4} \rho_{m} (U_{c} - U_{B})^{2} + 2k^{4} \rho_{m} \mu_{m} (U_{c} - U_{B}) D i - k^{4} \mu_{m}^{2} D^{2} =$$

$$= k \rho_{m} \left( 1 - \frac{\rho_{n} D}{\rho_{m} \kappa} \right) \left[ k^{3} \rho_{m} (U_{c} - U_{B})^{2} + 2\mu_{m} k^{3} D i (U_{c} - U_{B}) + \alpha k^{3} D \right]$$

We divide all terms of this equation by  $k^{\mbox{$ \over $}}\alpha\rho_{m}$  in order to put the equation in dimensionless form.

We thus obtain

$$\frac{\rho_{m} (U_{c} - U_{b})^{2}}{\alpha \kappa} + 2i \frac{\mu_{m} (U_{c} - U_{b}) D}{\alpha \kappa} - \frac{\mu_{m}^{2} D}{\alpha \kappa \rho_{m}} =$$

$$-\left(1 - \frac{\rho_{b} D}{\rho_{m} \kappa}\right) \left[\frac{\rho_{m} (U_{c} - U_{b})^{2}}{\alpha \kappa} + 2i \frac{\mu_{m} (U_{c} - U_{b})}{\alpha \kappa} + \frac{D}{\kappa}\right]$$

or

$$\frac{\rho_{_{\mathbf{B}}}}{\rho_{_{\mathbf{m}}}} \frac{D}{\kappa} \left[ \frac{\rho_{_{\mathbf{m}}} (U_{_{\mathbf{c}}} - U_{_{\mathbf{B}}})^{2}}{\alpha \kappa} + 2i \frac{\mu_{_{\mathbf{m}}} (U_{_{\mathbf{c}}} - U_{_{\mathbf{B}}}) D}{\alpha \kappa} \right] - \frac{\mu_{_{\mathbf{m}}}^{2} D^{2}}{\alpha \kappa \rho_{_{\mathbf{m}}}} - \left( 1 - \frac{\rho_{_{\mathbf{B}}} D}{\rho_{_{\mathbf{m}}} \kappa} \right) \frac{D}{\kappa} = 0.$$
(42)

Equation (42) qualitatively shows the presence of one unstable deformation, determined by the most hazardous initial disturbance. This initial disturbance develops more rapidly than the others, leading to the breaking away of a paddle wave from the surface of separation. This breaking away will occur for those values of the wave number k or the wavelength 1 for which the frequency of oscillation has a maximum value.

Solution of the equation for the initial disturbances with less idealized conditions of the problem, and an investigation of the roots of the characteristic equation, show (ref. 3) that there exist several unstable waves on the surface of separation.

It should be supposed that if we succeeded in investigating thoroughly the nonlinear equations of disturbances without any idealization, then we could obtain a large number of partial solutions corresponding to initial disturbances whose amplitudes increased with time.

As theoretical and experimental investigations show (refs. 1 and 4), with increased speed of motion, the number of unstable waves rapidly increases, and the surface of separation becomes such that there appear conditions for which the oscillations become unstable over a very wide range of variation of the wavelength.

For velocities close to the atomization mode, an infinitely large number of unstable waves will appear on the surface.

It is impossible to investigate thoroughly the appearance and development of the entire set of initial disturbances, due to the absence of the requisite mathematical apparatus. It is also impossible to pose the problem concretely, due to the indeterminacy of the initial conditions.

However, obtaining criteria which would permit us to determine the character of the decomposition, the coarseness of the atomization, and the spectrum of the distribution of drops by dimension, is possible, even with an idealized consideration of the conditions for instability given some disturbance.

For this we use equation (42), from which we can derive the following dimensionless parameters

1) 
$$\frac{\rho_{\rm B} D}{\rho_{\rm m} \kappa}$$
; 2)  $\frac{\rho_{\rm m} (U_{\rm c} - U_{\rm B})^2}{\alpha \kappa}$ ; 3)  $\frac{\mu_{\rm m} (U_{\rm c} - U_{\rm B}) D}{\alpha \kappa}$ ; 4)  $\frac{\mu_{\rm m}^2 D^2}{\alpha \kappa \rho_{\rm m}}$ .

It is convenient to replace parameters 3 and 4 by just one, which we obtain by dividing parameter 3 by parameter 4

3a) 
$$\frac{\rho_{m} (U_{c} - U_{b})}{\mu_{m} D}.$$

Parameters 1, 2 and 3a characterize an unstable deformation of the surface of separation.

We now turn to the establishment of criteria of the decomposition of a fuel film, flowing from a narrow port.

If we introduce into equation (42) the characteristic dimension of the film--its thickness--then this equation becomes the equation of unstable decomposition of a viscous fuel. If we denote by  $\delta$  the thickness of the film then, by introducing it into the dimensionless parameters 1, 2 and 3a, we obtain

1a) 
$$\frac{\rho_B}{\rho_m} \frac{D\delta}{\kappa \delta}$$
; 2a)  $\frac{\rho_m (U_c - U_B)^2 a}{\alpha} \frac{1}{\kappa \delta}$ ; 3b)  $\frac{\rho_m (U_c - U_B) a}{\mu_m} \frac{1}{D\delta}$ . (43)

The quantity D is proportional to the wave number k so that only one of these quantities should be introduced into the criteria. We now transform the quantity  $1/k\delta$  in the following way

$$\frac{1}{\kappa\delta} = \frac{1}{2\pi\delta}$$

In considering the decomposition of a jet, it is of interest to determine the distance from the output opening or port at which decomposition occurs.

By assuming that the length of the compact portion of the jet  $l_{\rm c}$  is proportional to the wave length 1 leading to decomposition, one can derive from the parameters of (43) the following criteria for the decomposition of a viscous fuel film

$$\frac{I_{c}}{\delta} = idem; \frac{\rho_{m}}{\rho_{m}} = idem; -\frac{\rho_{m}(U_{c} - U_{B})^{2} \delta}{\alpha} = idem;$$

$$\frac{\rho_{m}(U_{c} - U_{B})^{\delta}}{\mu_{m}} = idem. \tag{44}$$

On the basis of the  $\pi$ -theorem of similarity theory, there must exist among these criteria for similar processes a functional relationship of the form

$$\frac{I_c}{\delta t} = f\left[\frac{\rho_m (U_c - U_B)^2 a}{a}; \frac{\rho_m (U_c - U_B) a}{\mu_m}; \frac{\rho_m}{\rho_B}\right] \tag{45}$$

or, for fuel injection in stationary air,

$$\frac{I_{c}}{\delta} = f\left(\frac{\rho_{m} U_{c}^{2} \delta}{\alpha}; \frac{\rho_{m} U_{c} \delta}{\mu_{m}}; \frac{\rho_{m}}{\rho_{m}}\right). \tag{46}$$

Based on the consideration of the question of instability of the surface of separation, we can also obtain a functional relationship for the determination of the drop dimensions.

G. I. Petrov and T. D. Kalinina (ref. 1) assume that in the majority of cases the ultimate dimensions of the drops are determined directly from the conditions of decomposition of the basic jet, and that even in the case of subsequent granulation, the ultimate dimensions depend on the character of the deformation already developing in the jet prior to the ultimate decomposition.

Therefore, in the case of stationary air, we may write, for a representative dimension  $\textbf{d}_{\text{d}}$  of a drop

subscript 
$$K = d$$
 (drop) 
$$\frac{d_{\kappa}}{\delta} = f_1 \left( \frac{\rho_m \ U_c^2 \ a}{\alpha}; \frac{\rho_m \ U_c \ a}{\mu_m}; \frac{\rho_m}{\rho_n} \right). \tag{47}$$

We also obtained an analogous relationship (ref. 5) for a circular jet, on the basis of a consideration of the fundamental forces acting to atomize the jet. The experiments, and the processing of the data of other investigators, which we performed showed that the functional relationship of (47) correctly defines the granulation of the atomization.

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